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QUANTUM ERROR CORRECTION WITH A GLOBALLY-COUPLED ARRAY OF NEUTRAL ATOM QUBITS

UNIVERSITY OF WISCONSIN

FEBRUARY 2013

FINAL TECHNICAL REPORT

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14. ABSTRACT This is the final report for the FA8750-09-1-0230, "Quantum Error Correction with a Globally-Coupled Array of Neutral Atom Qubits." Under this project we developed and implemented an array of neutral atom qubits in optical traps for studies of quantum error correction. At the end of the three year project we were able to demonstrate preparation of a six qubit array in a two dimensional lattice with 4 micron site-to-site spacing. Numerical studies of quantum error correction (QEC) protocols were performed to determine the gate fidelity needed for useful error correction. A full experimental implementation of QEC is the subject of ongoing research in the Saffman labs at UW Madison.					
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1.0 SUMMARY

Under this project we developed and implemented an array of neutral atom qubits (quantum bits) in optical traps for studies of quantum error correction. At the end of the three year project we were able to demonstrate preparation of a six qubit array in a two dimensional lattice with 4 micron site-to-site spacing. Numerical studies of quantum error correction (QEC) protocols were performed to determine the gate fidelity needed for useful error correction. A full experimental implementation of QEC is the subject of ongoing research in the Saffman labs at UW Madison.

2.0 INTRODUCTION

Error correcting circuitry is a necessary requirement for scaling quantum computing devices to many qubits which execute multiple quantum gates as part of large scale quantum computations. Errors in quantum logic circuits can in principle be corrected by encoding logical qubits in several physical qubits. It can be shown [Nielsen2000] that the smallest number of physical qubits needed for correction of arbitrary errors is five. Many different encoding schemes have been proposed using five, seven, nine or larger numbers of physical qubits.

In general an arbitrary error in a qubit state can be decomposed into bit flip, phase flip, or combined bit- and phase-flip errors. Experiments performed in the last few years have demonstrated correction of bit flip or phase flip errors [Chiaverini2004, Schindler2011, Reed2012]. To date there has been no demonstration of a complete QEC protocol capable of correcting arbitrary qubit errors. Such a demonstration will be a major step forwards towards practical quantum computing.

An array of neutral atom qubits is a promising approach to implementing a complete QEC protocol. The long range nature of Rydberg interactions [Saffman2010] will enable gates between arbitrary pairs of qubits in a globally coupled logical qubit. The Rydberg blockade interaction also allows for efficient multi-qubit gates [Isenhower2011, Mølmer2011] which may prove useful for QEC implementations. In this project we have developed and implemented a new type of optical lattice based on a Gaussian beam array (GBA) [Piotrowicz2013]. The GBA traps ground and Rydberg states at local minima of the optical intensity. The ability to trap the Rydberg states which are used for entangling gates is new [Zhang2013] and is expected to lead to enhanced fidelity of Rydberg gate implementations.

In addition to experimental activities we have addressed several theoretical issues related to QEC with neutral atoms. First, we have performed a rigorous calculation of the best obtainable fidelity of an entangling Rydberg C_Z gate [Zhang2011]. This calculation established an error floor below 0.002 which is compatible with scalable quantum computation. We have also studied numerically the gate fidelity needed for implementation of QEC [Maller2013].

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Experimental platform

Our experiments are based on optical trapping of laser cooled Cs atoms. We use a double UHV system. In a primary vacuum region a 2D source of cold atoms is prepared using standard laser cooling techniques. The cold atoms are then transferred through a 1 mm aperture into a six sided optical “science” cell using a resonant 852 nm pushing beam. The science cell manufactured by

ColdQuanta (Boulder, Colorado) is constructed from pyrex with optically contacted sides. This cell provides close to 360 deg. optical access which is convenient for introducing multiple laser cooling, as well as qubit control and measurement beams.

The GBA uses 780 nm trapping light generated by a high power source laser system we have developed. The source laser based on a 1560 nm diode, fiber amplifier and frequency doubling stage [Lichtman2013] gives more than 10 W continuous output power at 780 nm. The 780 nm light is fiber-coupled to an optical system that splits the power into 16 weakly overlapping Gaussian beams as shown in Figure 1.

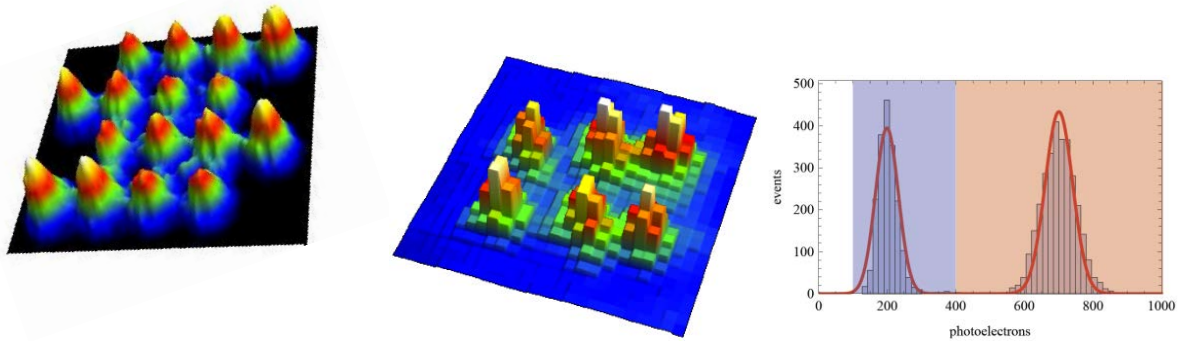


Figure 1. Atom trapping and detection in a 2D array. Intensity distribution of GBA lattice (left), fluorescence image of six atoms trapped in the array (center) and histogram showing high fidelity detection of a single atom (right).

Atoms are loaded into the array from a MOT (magneto-optical trap) located at the center of the science cell. Fluorescence from trapped atoms is imaged onto an intensified CCD camera (EMCCD) and recorded by a computer. The image in Fig. 1 (left) shows six atoms trapped in a 2D array with site-to-site spacing of 4 microns. The pixel size in the image is about 0.6 microns. These atoms are all close enough together that Rydberg-mediated quantum gating can in principle be performed between all qubits [Saffman2010, Saffman2011].

3.2 State preparation and logic operations

Atoms are prepared in a fiducial state $|f, m\rangle = |4, 0\rangle$ using optical pumping. The optical pumping is performed with 894 nm light z-polarized coupling $6s_{1/2}$, $f=4$ - $6p_{1/2}$, $f=4$, together with 852 nm light that is circularly polarized and couples $6s_{1/2}$, $f=3$ - $6p_{3/2}$, $f=4$.

Logic operations are performed by focusing control laser beams onto individual sites. Beam scanning is performed using 2D acousto-optic scanners constructed from crossed 1D acousto-optic devices. These scanners have been verified to cover a 5x5 array of 25 sites with switching time of about 100 ns. Scanners have been constructed for 459 nm light which is used for single qubit gates and Rydberg excitation, 1038 nm light which is used for Rydberg excitation, and 852 nm light which is used for qubit readout.

Single qubit gates use 459 nm light near resonance with the Cs $6s_{1/2} - 7p_{1/2}$ transition. The light is detuned from the transition by about 50 GHz and contains two frequency components to drive transitions between the qubit clock states $|f, m\rangle = |3, 0\rangle$ and $|4, 0\rangle$ which are separated by 9.2 GHz.

Two-qubit entangling gates are to be performed via Rydberg excitation as has been demonstrated earlier using Rb atoms. Details of the Rydberg blockade gate protocol and implementation can be found in [Isenhower2010, Zhang2010, Walker2012]. The excitation scheme used for Cs is based on two-photon excitation with 459 + 1038 nm light coupling the ground state to a Rydberg ns or nd level via $7p_{1/2}$.

4.0 RESULTS AND DISCUSSION

4.1 Process tomography of single qubit gates

As a precursor to experiments in the array we have verified the operations of state preparation and single qubit control by performing process tomography of single qubit gates. We demonstrate three-axis control of the qubit. Those experiments used a setup with a single trapping site. Details of the experimental setup are given in [Li2012]. Process tomography fidelity can be characterized using different measures. As we have discussed in [Zhang2012] different measures capture different types of gate errors (Figure 2). The results shown in the table include errors due to state preparation and state measurement. Ongoing work is directed towards improving the gate fidelity to above 0.95 for QEC experiments.

$$F_O^{1/2} = \text{Tr}[\sqrt{\sqrt{\chi_{sim}} \chi_{id} \sqrt{\chi_{sim}}}]$$

$$F_O = \text{Tr}^2[\sqrt{\sqrt{\chi_{sim}} \chi_{id} \sqrt{\chi_{sim}}}]$$

$$F_D = 1 - \frac{1}{2} \text{Tr}[\sqrt{(\chi_{sim} - \chi_{id})^+ \chi_{sim} - \chi_{id}}]$$

Gate	$F_O^{1/2}$	F_O	F_D
$R_x(\pi)$	0.83	0.69	0.67
$R_x(\pi/2)$	0.92	0.85	0.84
$R_y(\pi)$	0.90	0.80	0.79
$R_y(\pi/2)$	0.90	0.81	0.76
$R_z(\pi/4)$	0.88	0.77	0.76
average	0.89	0.78	0.76

Figure 2. Single qubit process tomography. Fidelity results are given for x,y and z gates using three different fidelity measures.

4.2 Rydberg excitation of trapped atoms

We have demonstrated Rabi oscillations of single trapped Cs atoms between ground and Rydberg states. The oscillations used two-photon excitation via the second resonance level of Cs. Specifically the scheme was $6s_{1/2}, f=4, m=0 \rightarrow 7p_{1/2} \rightarrow 61d_{3/2}$. The lasers were 459 nm for the first leg and 1038 nm for the second leg.

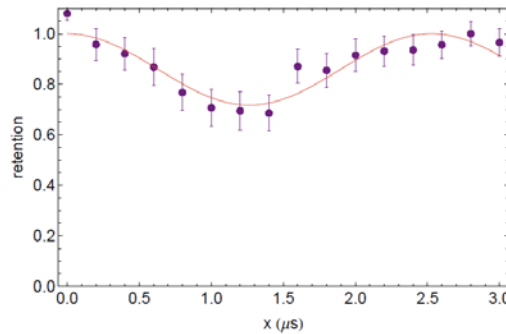


Figure 3. Rydberg Rabi oscillations. The y-axis shows the ground state population, which reaches a minimum at 1.3 microsec. due to Rydberg excitation.

This data shows the ability to drive Rabi oscillations with both ground and Rydberg atoms inside the same optical trap. This is the first time this has been done using the 2nd resonance level (7p1/2) in Cs. This is significant since it allows us to use a high power fiber laser for the 1038 nm leg which will be important for scaling to multiple sites and fast Rabi oscillations which are needed for high fidelity entangling gates. Note that the relatively small amplitude of the oscillation in Figure 3 is due to the fact that detection is based on Rydberg atom loss and we are using a scheme with minimized loss (see next section). Accounting for this fact we estimate the underlying oscillation amplitude at better than 80%.

4.3 Trapping Rydberg excited atoms

The Rydberg blockade entangled experiments that have been performed to date [Isenhower2010, Wilk2010] used red detuned optical traps where the atoms are localized at a local maximum of the trap light intensity. Rydberg excitation in a red trap leads to rapid loss of the Rydberg atom due to photoionization and repulsive mechanical forces since the Rydberg state has a negative polarizability (the optical interaction is essentially that of a free electron). These problems were mitigated by turning off the traps during the Rydberg excitation sequence. While this works at the level of two atoms it is problematic for array operations and for performing multiple gates in sequence due to unwanted atomic heating and loss from turning the traps off.

To solve this problem we have proposed using blue detuned traps for both ground and Rydberg state atoms [Zhang2011]. This will obviate the need for turning the array off during each Rydberg pulse and is essential to reducing unwanted entanglement between spin (qubit) and center of mass degrees of freedom, as well as collateral heating on qubits in the array. Proper design of the trap size allows for magic trapping between ground and Rydberg states which will also improve the coherence of Rydberg excitation. Under this program we have experimentally demonstrated, for the first time, the ability to trap Rydberg excited atoms in 3D [Zhang2013].

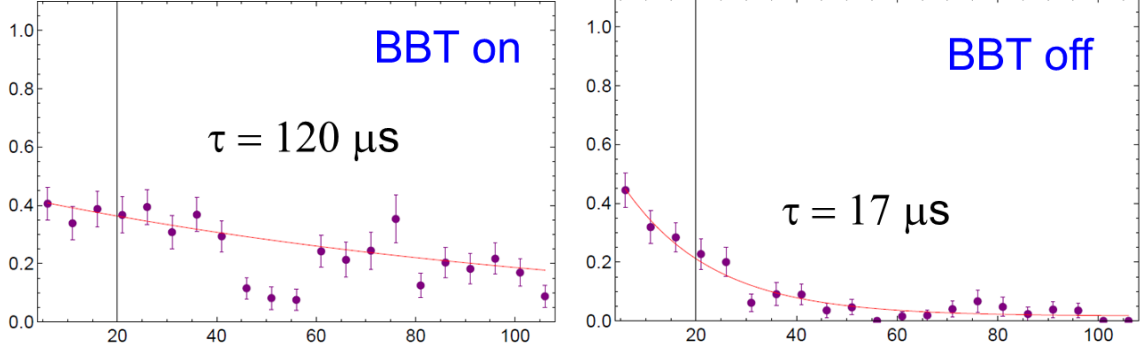


Figure 4. Optical trapping of Rydberg atoms. The atom lifetime with BBT on (left) is much longer than the lifetime with BBT off (right).

Rydberg trapping experiments were performed in the setup described in [Li2012]. The data in Figure 4 show trap lifetime curves for a Rydberg excited atom with the Bottle Beam trap (BBT) on (left) and off (right). Data were taken for a Cs atom excited to $61d_{3/2}$. The x7 increase in lifetime is clear evidence for Rydberg state trapping. In other experiments we have confirmed a trap induced Rydberg transition shift of ~ 100 kHz, so we are close to the ideal magic condition.

In order to estimate the loss rate of the trapped Rydberg atom we compare the measured lifetime $\tau_{\text{meas}} = 120$ microsec., with the time constant for a Rydberg atom to return to the ground state. Using a Monte Carlo simulation of the Rydberg atom decay we estimate that time constant to be $\tau_{\text{rad}} = 143$ microsec. This implies a trap lifetime given by

$$\frac{1}{\tau_{\text{meas}}} = \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_{\text{trap}}}. \quad (1)$$

Using the measured τ_{meas} and calculated τ_{rad} we find $\tau_{\text{trap}} = 750$ microsec. Since this time is several orders of magnitude longer than Rydberg gate times we can effectively eliminate Rydberg atom loss during gate operation using this kind of trap.

4.4 Gate fidelity and QEC

In an ideal system where gates are performed with perfect fidelity known methods of QEC [Nielsen2000] can be used to correct for unwanted errors that affect qubit states. Errors are expected to occur due to technical imperfections, but also from coupling to the environment which can never be fully eliminated. It would be unrealistic to assume that errors only affect quantum states and not quantum operations, such as gates. With finite fidelity gates the successful implementation of QEC and a scalable quantum processor becomes more challenging. Threshold theorems show that if gate errors are not too large then it will be possible to scale a quantum computer to arbitrary size using additional resources that do not scale exponentially, thereby maintaining an advantage over classical computing systems.

The threshold fidelity needed to construct a scalable system depends on what assumptions are made about errors and system resources. Roughly speaking there is a trade-off between the

number of additional qubits needed for encoding against errors, and the tolerable error threshold. While there are calculations showing error thresholds above 10%, the corresponding architecture invokes very large numbers of ancillas. Encodings with five or seven qubits generally require errors at the 0.005 – 0.0001 level to be feasible, again depending upon architecture used.

Since the best entanglement fidelity demonstrated to date using Rydberg blockade is around 75% [Zhang2010, Wilk2010] it has been important to ascertain what the theoretical error limit is for a Rydberg blockade gate. This was done by us in [Zhang2012] using a rigorous calculation based on master equation solutions to simulate quantum process tomography. Rydberg states were identified with process fidelity errors <0.002 . This establishes that the Rydberg blockade gate is a viable approach for continued research towards a scalable quantum processor.

In the intermediate term it is also important to understand how good our gates need to be to observe a fidelity benefit from applying QEC with imperfect gates. We have looked at this numerically starting with a three qubit encoding to correct bit flips. Figure 5 shows measurement based and coherent circuits for correcting bit flip errors.

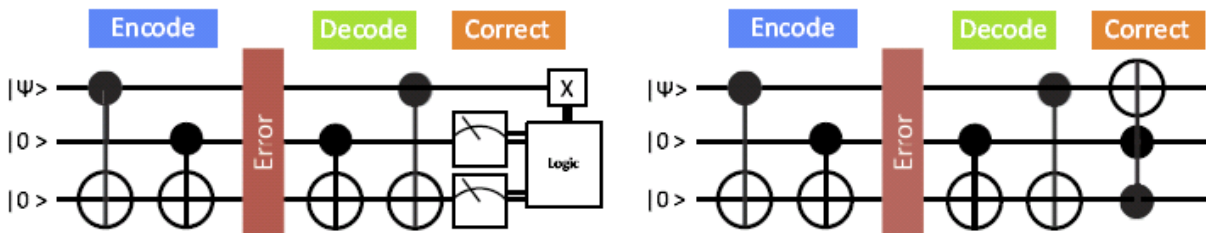


Figure 5. QEC circuits for bit flip errors. Measurement based QEC (left) and coherent QEC (right).

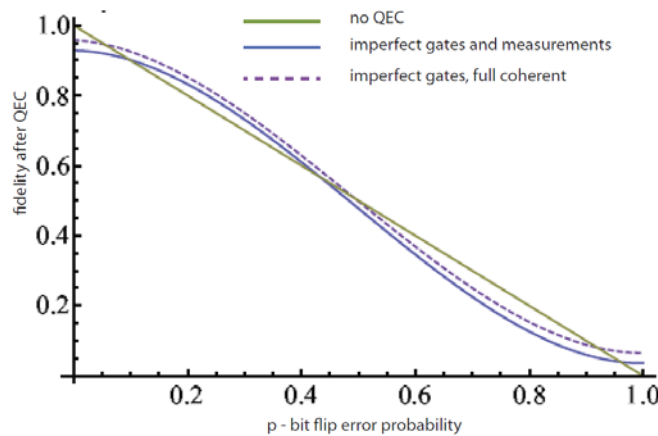


Figure 6. Numerical simulation of performance of measurement based and coherent QEC.

In Figure 6 we show the results of numerical simulations with imperfect gates used in the error correction circuits. The straight line shows the decay of the state fidelity when subject to a bit flip with error probability p . The calculation started with state $|0\rangle$. The solid curve shows the result of measurement based QEC with gate fidelity 0.988 and measurement fidelity 0.96. The dashed curve shows the result of coherent QEC with two-qubit gate fidelity 0.988 and Toffoli gate fidelity (the last step in Fig. 5) of 0.95.

We see that the coherent approach has slightly better performance for the parameters chosen. However this conclusion needs to be verified for a wider range of gate and measurement error values. This is being studied in ongoing work [Maller2013] which is also looking at the performance of a full QEC implementation using the 5 bit code. In either case gates with fidelity ~ 0.99 are needed to observe a positive effect of QEC.

5.0 CONCLUSIONS

In the framework of this project we have made substantial progress towards QEC with neutral atom qubits. We have demonstrated trapping of neutral atom qubits in a site-resolved 2D array. We have performed quantum gates on single atom qubits and have demonstrated trapping of Rydberg excited atoms. In ongoing work we are combining these experimental capabilities with the goal of a near term demonstration of entangling gates in the 2D neutral atom qubit array.

We have theoretically verified that the Rydberg blockade interaction can be used for gates with 0.002 errors and studied the performance of QEC circuits with plausible gate fidelities. The publications which have appeared in peer reviewed journals resulting from support under this project are [Isenhower2011, Li2012, Mølmer2011, Saffman2010, Walker2012, Zhang2011, Zhang2012]. In addition there are four unfinished manuscripts [Lichtman2013, Maller2013, Piotrowicz2013, Zhang2013] which will be submitted in 2013 and will acknowledge DARPA support.

In the framework of this project there was not sufficient time to fully complete the planned experiments demonstrating QEC. We are continuing to work towards a full demonstration of QEC with support from other funding sources.

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7.0 LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

1D	1 Dimensional
2D	2 Dimensional
3D	3 Dimensional
BBT	Bottle beam trap
GBA	Gaussian beam array
EMCCD	electron multiplying charge coupled device
microsec.	microsecond
MOT	Magneto-optical trap
QEC	quantum error correction
qubit	quantum bit